# A STUDY OF BETA CEPHEI

A Dissertation Submitted in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy in the
University of Michigan

By

**CLIFFORD C. CRUMP**

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# A STUDY OF BETA CEPHEI[1]

By CLIFFORD C. CRUMP

## INTRODUCTION

The study of a spectroscopic binary of very short period is of especial interest because of the material it affords for the furtherance of our knowledge of stellar evolution. Through the observation of such a system we are able to derive data with reference to the development of binaries in their early stages and to investigate the laws which govern their evolution. If a star in addition to being a short period binary be a variable of the δ Cephei type the interest is enhanced, for while there are considerable data and many theories concerning Cepheid stars, astronomers agree that a satisfactory solution of the problem of their variation has not been reached and all contributions to the facts already known are particularly valuable.

It is the purpose of this paper to give the results of the study of β Cephei, which is both a short period binary and a variable of the δ Cephei type. The period of β Cephei is the shortest that has thus far been determined for a spectroscopic binary, namely, 4 hours 34 minutes and 17.4 seconds, enabling one to follow it for more than a revolution in a single night. It is remarkable that plates taken with the time of mid-exposure only six minutes apart show a measurable change in the radial velocity of this star. The shortness of the period and the brightness of the star, 3.3 magnitude, make it possible to determine in a comparatively short time the definitive elements of the orbit with the observations grouped around a well defined epoch. It is also possible to compare readily complete cycles of the curve. The recent investigations of the light variation of β Cephei by Guthnick of the Berlin-Babelsberg Observatory have added to the importance of a complete spectroscopic study of this star.

It is proposed to determine definitively the orbital elements of β Cephei and to bring together the existing data concerning it.

[1] A dissertation in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the University of Michigan.

## TOTAL LIGHT

As a result of the photoelectric measures of the light variation of β Cephei, Guthnick has announced the star to be a variable. He reaches this conclusion from a series of observations which extend over six months and comprise one hundred and seventy-seven determinations of the light intensity. The light curve plotted from the normal places of his observations indicates an amplitude of 0.05 magnitude as measured with the sodium cell and a range in time from maximum to minimum light of 0.09 days, a little more than one-half the spectroscopic period. The light curve determined by Guthnick compared with the radial velocity curve as observed by Frost showed, on account of the position of the points of maximum and minimum light, that the star was not an Algol nor a β Lyræ variable. The general form of the light curve and the degree of agreement of the point of maximum light with the point of maximum velocity of approach led to the assumption that this variable was of the δ Cephei type. Plate IX, Figure 2, shows the light curve plotted from Guthnick's observations, which may easily be compared with Figure 1 of the same plate, which shows the velocity curve determined at the Detroit Observatory.

According to the general characteristics of Cepheid variables the maximum of light should coincide approximately with the maximum velocity of approach. In most cases this agreement lies within one-fifteenth of the period. Here we observe the difference in time to be 0.029 days, or about one-seventh of the period, a quantity approximately equal to that observed in the case of the Cepheid variable, Y Sagittarii, but with light maximum occurring on the descending branch instead of the ascending branch of the velocity curve. In β Cephei the variation from the general rule of close coincidence between light maximum and maximum velocity of approach may be connected with the closeness of the components of the system. The correspondence between the minimum of light and the mini-

mum velocity of approach in the cases so far observed has not been so close on the average as that between the maximum of light and the maximum velocity of approach. The discrepancy between light minimum and the maximum velocity of recession in the case of $\beta$ Cephei, though it amounts to about one-fifth of the period, should not exclude it from the $\delta$ Cepheid class. This disagreement may be due here and in other cases to variations caused by the ellipsoidal form of the members of the system.

### HISTORICAL

From December 18, 1901, to April 16, 1902, six spectrograms of $\beta$ Cephei ($\alpha = 21^h\ 27^m$, $\delta = +70^\circ\ 7'$) were obtained by Mr. W. S. Adams with the Bruce spectrograph of the Yerkes Observatory. The measures of the third plate of this series suggested a variable radial velocity for this star. This suggestion was confirmed by subsequent investigations of Professor Edwin B. Frost and the binary character of the star was established. An early set of radial velocity determinations showed a range of from — 20.3 km. to + 11.3 km. The period was at first thought to be rather long, but Professor Frost, from two spectrograms taken on the night of May 14, 1902, during an interval of five and one-half hours, observed a change in velocity of 14 km. and inferred that the period was short. Owing to the insufficient data at hand Professor Frost at that time made no attempt to draw further conclusions.

In the spring of 1906 the problem was again taken up by Professor Frost with the idea of determining the period. On the nights of May 18, 19, 20, and 21, eight plates were secured. The approximate measures of these plates indicated that the period was about four hours and thirty minutes. The star was continuously observed through several nights, and from May 28 to August 27 more than sixty plates were made. From the observations of July 6, the following elements were derived:

ELEMENTS OF JULY 6, 1906.

$P = 0.190479$ days,
$e = 0.03 +$,
$\omega = 165^\circ\ 13'$,
$T = 20^h\ 37^m$ G. M. T., or J. D.2417398.859,
$K = 16.9$ km.,
$\gamma = -4.9$ km.

At the meeting of the National Academy of Sciences in December, 1914, Professor Frost gave a brief discussion of this system. In a letter which I have recently received he gives the value of $\gamma$ on the night of April 29, 1912, as — 10.2 km. The discrepancy between the velocity of the center of mass as given for July 6, 1906, and that determined for this later date is of particular significance and will be discussed in connection with the elements derived by the writer.

### OBSERVATIONS

Observations of $\beta$ Cephei were begun at the Detroit Observatory on August 8, 1912, and were continued until November 21, of that year. During this interval 175 spectrograms were made with the one-prism spectrograph attached to the 37½-inch Reflector. Of the total number of spectrograms obtained 163 were used in the determination of the radial velocities. For the most part Seed 23 plates were used. On the night of October 15 Seed 30 plates were used, with the exception of plates 1247 B, 1249 C, 1250 D, and 1269 B, which were made on Seed 23 plates. The time of exposure varied from five to fifteen minutes. On November 7, conditions being very favorable, thirty-six plates were secured with an average exposure of six minutes. The last series of observations, November 21, was taken with a narrow slit for the purpose of searching the spectrum for double lines. It was the aim from the beginning to follow the star continuously through the period assumed by Frost and on four different nights we were successful in this endeavor.

### SPECTRUM

The spectrum of $\beta$ Cephei is of the Orion type. It is designated in the *Harvard Annals*, Vol. 50, by B1, and in Miss Maury's classification in the *Harvard Annals*, Vol. 28, by IIIa. The chief lines of the spectrum are due to helium, hydrogen, oxygen, and silicon. For a spectrum of this class the lines are well defined and on the whole easy to measure. The series of plates taken with a narrow slit for the purpose of detecting a second component's spectrum gave no evidence that such a spectrum existed. The region used in this investigation extended from $\lambda$ 3933 to $\lambda$ 4922. The comparison spectrum was that of the titanium spark.

### MEASUREMENT

A part of the spectrograms, namely those of the night of November 21, were measured with Measuring Engine No. 1 of this Observatory, and the remainder of the plates were measured at the Observatorio Nacional de La Plata, Argentina, with a Gærtner engine belonging to the Cordoba Observatory. The screw error of both these engines was so small that correction of the micrometer readings was unnecessary. Twenty-two comparison lines were used on each plate. Three settings were made on the upper comparison line and three on the lower, and the average of these readings was taken as the reading for that line. The average of three settings on the star line was taken as the reading for this line. The correction for curvature of the spectral lines, although small, was taken into account in the reductions. All of the plates were measured direct and reversed in order to eliminate personal equation as far as possible.

### WAVE-LENGTHS

Twenty-two star lines were selected for use in the determination of the radial velocities. The wave-lengths were taken from published tables containing such quantities and are incorporated in Table I. Wherever a line was used which could not be identified in some published table the wave-length was computed from the well known Hartmann Formula,

$$\lambda = \lambda_0 + \frac{c}{R_0 - R}.$$

The constants of this formula for the spectrograph used had previously been determined by Mr. Mellor, whose results are published on Volume 1, page 140, of the *Publications of the Detroit Observatory*. The standard of dispersion used by Mr. Mellor having been adopted, the constants determined by him have been used in these reductions.

### RADIAL VELOCITIES

The velocities were determined from the measures in the usual manner. Corrections were applied for the orbital velocity of the earth, for the earth's rotation, for the curvature of the lines, and for the small increments which were applied to the wave-lengths of the several star lines to make them homogeneous.

TABLE I. WAVE-LENGTHS OF LINES MEASURED.

| ASSUMED WAVE LENGTH. | WAVE-LENGTH IN $\beta$ CEPHEI. | AUTHORITY. | ELEMENT. |
|---|---|---|---|
| Å | Å | | |
| 3933.83 | 3933.78 | Rowland. | K, Calcium. |
| 3964.88 | 3964.84 | Runge and Paschen. | Helium. |
| 3968.62 | 3968.65 | Rowland. | H$\epsilon$, Hydrogen. |
| 3973.48 | 3973.40 | Eisig. | Oxygen. |
| 4009.12 | 4009.16 | Runge and Paschen. | Helium. |
| 4026.37 | 4026.37 | Runge and Paschen. | Helium. |
| 4070.05 | 4069.94 | Crump. | |
| 4072.16 | 4072.06 | Crump. | |
| 4076.08 | 4076.01 | Exner and Haschek. | Oxygen. (?) |
| 4089.00 | 4089.16 | Lunt. | Silicon. |
| 4101.92 | 4101.92 | Curtiss. | H$\delta$, Hydrogen. |
| 4121.02 | 4120.92 | Runge and Paschen. | Helium. |
| 4143.92 | 4143.93 | Runge and Paschen. | Helium. |
| 4253.81 | 4254.01 | Crump. | |
| 4267.15 | 4267.22 | Eder and Valenta. | Carbon. |
| 4340.63 | 4340.64 | Rowland. | H$\gamma$, Hydrogen. |
| 4388.10 | 4388.06 | Runge and Paschen. | Helium. |
| 4471.68 | 4471.65 | Runge and Paschen. | Helium. |
| 4552.76 | 4552.82 | Albrecht. | Silicon. |
| 4713.31 | 4713.53 | Runge and Paschen. | Helium. |
| 4861.53 | 4861.56 | Rowland. | H$\beta$, Hydrogen. |
| 4922.10 | 4922.10 | Runge and Paschen. | Helium. |

The correction for curvature of the spectral lines, which was obtained on the assumption that the lines were parabolic in form, was made from measures of well defined lines. This correction although small was applied to each determination.

The correction for making the lines homogeneous was made in the following manner. Residuals were found from a comparison of the velocity as given by the line and the velocity as given by the plate. A correction was then applied to each line which would reduce to zero the algebraic sum of the residuals of that line for all of the plates on which it was measured.

The method used in determining the final weights for the lines has been described by Professor Curtiss in his article on $\delta$ Orionis, in the *Publications of the Detroit Observatory*, Volume 1, page 123.

Table II contains the observed velocity data. Column 1 gives the number of the plate; column 2, the time of mid-exposure reckoned in days from January 0.0, 1912; column 3, the phase computed with the period determined below, 0.1904795 days, and referred to the epoch of the first observation, 1912, August 23,7576 G.M.T.; column 4, the number of lines measured; column 5, the weight assigned to the plate; column 6, the radial velocity; column 7, the residuals from the final elliptic elements; and column 8, general remarks.

TABLE II. ANN ARBOR OBSERVATIONS.

| PLATE NO. | G. M. T. 1912. | PHASE. | NO. OF LINES. | WT. | VELOCITY. | O–C | REMARKS. |
|---|---|---|---|---|---|---|---|
| | | | | | km. | km. | |
| 1019 D | 236.7576 | 0.0000 | 15 | 13 | —21.07 | —3.6 | |
| 1023 B | 239.7847 | 0.1699 | 15 | 11.5 | —19.65 | +6.2 | |
| 1027 B | 239.8321 | 0.0268 | 17 | 12.5 | — 0.33 | +3.7 | |
| 1028 C | 239.8448 | 0.0395 | 17 | 14 | — 2.50 | —2.3 | |
| 1029 A | 239.8601 | 0.0548 | 11 | 6.5 | + 2.84 | +0.6 | |
| 1035 A | 240.6979 | 0.1307 | 19 | 15 | —27.86 | —0.8 | |
| 1038 C | 242.6437 | 0.1717 | 19 | 22 | —23.58 | +1.7 | |
| 1039 D | 242.6528 | 0.1808 | 21 | 31 | —27.72 | —6.1 | |
| 1040 A | 242.6618 | 0.1898 | 19 | 25.5 | —22.05 | —4.6 | |
| 1041 B | 242.6799 | 0.0174 | 18 | 18.5 | —11.55 | —3.2 | |
| 1042 C | 242.6896 | 0.0271 | 17 | 10.5 | — 5.61 | —1.9 | |
| 1043 D | 242.6999 | 0.0374 | 17 | 19 | — 1.38 | —1.3 | |
| 1044 A | 242.7125 | 0.0500 | 17 | 16.5 | — 0.52 | —2.8 | |
| 1045 B | 242.7243 | 0.0618 | 19 | 18.5 | — 2.11 | —3.6 | |
| 1116 B | 275.5646 | 0.1397 | 15 | 26.5 | —36.32 | —7.5 | |
| 1117 C | 275.5736 | 0.1487 | 17 | 21.5 | —34.73 | —5.4 | |
| 1118 D | 275.5833 | 0.1584 | 17 | 41 | —34.23 | —5.7 | |
| 1119 A | 275.5924 | 0.1675 | 17 | 36.5 | —29.94 | —3.3 | |
| 1120 B | 275.6000 | 0.1751 | 16 | 29 | —25.86 | —1.9 | |
| 1121 C | 275.6090 | 0.1841 | 15 | 23 | —16.51 | +3.6 | Poor. |
| 1122 D | 275.6159 | 0.0005 | 13 | 18.5 | —16.54 | +0.6 | |
| 1123 A | 275.6250 | 0.0096 | 17 | 24.5 | — 9.43 | +3.0 | |
| 1124 B | 275.6317 | 0.0163 | 12 | 16.5 | —10.77 | —1.9 | |
| 1125 C | 275.6410 | 0.0256 | 14 | 34.5 | — 3.39 | +1.0 | |
| 1126 D | 275.6486 | 0.0332 | 16 | 31 | + 3.08 | +4.4 | |
| 1127 A | 275.6562 | 0.0408 | 16 | 23.5 | + 5.56 | +4.7 | |
| 1128 B | 275.6639 | 0.0485 | 20 | 24 | + 9.51 | +7.4 | |
| 1129 C | 275.6722 | 0.0568 | 18 | 31.5 | + 9.90 | +7.7 | |
| 1130 D | 275.6771 | 0.0617 | 18 | 31 | +10.13 | +8.6 | |
| 1131 A | 275.6875 | 0.0721 | 16 | 38 | — 0.43 | +0.8 | |
| 1132 B | 275.6969 | 0.0815 | 18 | 38.5 | — 8.67 | —3.6 | |
| 1133 C | 275.7049 | 0.0895 | 18 | 27.5 | — 7.15 | +1.8 | |
| 1134 D | 275.7130 | 0.0976 | 15 | 25 | —16.10 | —3.0 | |
| 1135 A | 275.7211 | 0.1057 | 17 | 29.5 | —11.69 | +5.4 | |
| 1136 B | 275.7285 | 0.1131 | 19 | 22 | —16.35 | +4.4 | |
| 1137 C | 275.7361 | 0.1207 | 16 | 26 | —21.34 | +2.6 | |
| 1138 D | 275.7437 | 0.1283 | 15 | 18.5 | —27.31 | —0.8 | |
| 1139 A | 275.7521 | 0.1367 | 18 | 25 | —32.75 | —4.4 | |
| 1140 B | 275.7604 | 0.1450 | 18 | 26 | —28.63 | +0.5 | |
| 1141 C | 275.7705 | 0.1551 | 19 | 23 | —23.40 | +5.6 | |

TABLE II. ANN ARBOR OBSERVATIONS—Continued

| PLATE NO. | G. M. T. 1912. | PHASE. | NO. OF LINES. | WT. | VELOCITY. | O–C. | REMARKS |
|---|---|---|---|---|---|---|---|
| 1142 D | 275.7802 | 0.1648 | 18 | 27.5 | —23.56 | +3.7 | |
| 1247 B | 289.5579 | 0.0375 | 15 | 38 | + 5.10 | +5.1 | |
| 1248 A | 289.5642 | 0.0438 | 15 | 22.5 | + 0.55 | —0.9 | |
| 1249 C | 289.5712 | 0.0508 | 18 | 27 | — 0.48 | —2.7 | |
| 1250 D | 289.5802 | 0.0598 | 18 | 35 | + 5.52 | +3.8 | |
| 1251 A | 289.5885 | 0.0681 | 14 | 19 | — 2.33 | —2.3 | |
| 1252 B | 289.5962 | 0.0758 | 13 | 14.5 | — 2.19 | +0.4 | Poor. |
| 1253 C | 289.6017 | 0.0813 | 15 | 33 | — 0.08 | +4.9 | |
| 1254 D | 289.6066 | 0.0862 | 14 | 26.5 | — 5.41 | +2.0 | |
| 1255 A | 289.6121 | 0.0917 | 16 | 22.5 | — 8.82 | +1.3 | |
| 1256 B | 289.6194 | 0.0990 | 17 | 36.5 | —15.41 | —1.4 | |
| 1257 C | 289.6302 | 0.1098 | 14 | 24 | —22.06 | —2.8 | |
| 1258 D | 289.6351 | 0.1147 | 16 | 29 | —17.73 | +3.7 | |
| 1259 A | 289.6392 | 0.1188 | 14 | 14.5 | —23.99 | —0.7 | |
| 1260 B | 289.6434 | 0.1230 | 15 | 22.5 | —32.51 | —7.7 | |
| 1261 C | 289.6479 | 0.1275 | 11 | 15.5 | —33.48 | —7.3 | Poor. |
| 1262 D | 289.6521 | 0.1317 | 15 | 16.5 | —32.22 | —4.8 | |
| 1263 A | 289.6562 | 0.1358 | 11 | 14 | —29.19 | —0.9 | Poor. |
| 1264 B | 289.6604 | 0.1400 | 11 | 23.5 | —27.23 | +1.6 | |
| 1265 C | 289.6646 | 0.1442 | 13 | 15 | —23.30 | +6.0 | |
| 1266 D | 289.6694 | 0.1490 | 15 | 18 | —33.49 | —4.2 | |
| 1267 | 289.6748 | 0.1544 | 12 | 18 | —25.20 | +3.8 | |
| 1268 A | 289.6794 | 0.1590 | 13 | 24 | —36.76 | —8.4 | |
| 1269 B | 289.6854 | 0.1660 | 19 | 48 | —20.25 | +6.8 | |
| 1288 A | 291.6958 | 0.0801 | 15 | 22.5 | — 8.56 | —4.0 | |
| 1289 B | 291.7062 | 0.0905 | 19 | 34.5 | — 3.78 | +5.8 | |
| 1290 C | 291.7152 | 0.0995 | 15 | 25 | —13.11 | +1.0 | Poor. |
| 1291 D | 291.7243 | 0.1086 | 15 | 12 | —11.59 | +7.0 | Poor. |
| 1292 A | 291.7333 | 0.1174 | 15 | 20 | —18.28 | +4.3 | |
| 1293 B | 291.7424 | 0.1267 | 20 | 15.5 | —28.21 | —2.2 | Poor. |
| 1294 C | 291.7514 | 0.1357 | 18 | 18 | —28.73 | —0.5 | |
| 1295 D | 291.7611 | 0.1454 | 18 | 22 | —28.66 | +0.6 | |
| 1296 A | 291.7701 | 0.1544 | 20 | 17.5 | —20.71 | +8.3 | |
| 1297 B | 291.7792 | 0.1635 | 18 | 18 | —29.41 | —1.9 | |
| 1298 C | 291.7882 | 0.1725 | 16 | 18.5 | —21.64 | +3.5 | |
| 1299 D | 291.7972 | 0.1815 | 16 | 18.5 | —14.42 | +7.0 | |
| 1304 D | 298.6111 | 0.1381 | 17 | 31.5 | —29.19 | +0.5 | |
| 1305 A | 298.6229 | 0.1499 | 17 | 24.5 | —29.29 | +0.0 | |
| 1306 B | 298.6340 | 0.1610 | 17 | 26 | —26.54 | +1.5 | |
| 1307 C | 298.6414 | 0.1684 | 18 | 27.5 | —24.41 | +1.9 | |
| 1308 D | 298.6479 | 0.1749 | 16 | 26.5 | —21.23 | +3.0 | |
| 1309 A | 298.6555 | 0.1825 | 18 | 25 | —23.94 | —3.0 | |
| 1310 B | 298.6590 | 0.1860 | 16 | 15 | —16.32 | +3.0 | Poor. |
| 1311 C | 298.6653 | 0.0019 | 16 | 23.5 | —10.85 | +6.1 | |
| 1312 D | 298.6705 | 0.0071 | 17 | 29.5 | —11.21 | +2.5 | |

TABLE II. ANN ARBOR OBSERVATIONS—Continued.

| PLATE NO. | G. M. T. 1912. | PHASE. | NO. OF LINES. | WT. | VELOCITY. | O–C. | REMARKS. |
|---|---|---|---|---|---|---|---|
| | | | | | km. | km. | |
| 1313 A | 298.6746 | 0.0112 | 20 | 27.5 | —10.13 | +1.4 | |
| 1314 B | 298.6790 | 0.0156 | 19 | 24.5 | — 2.97 | +6.3 | |
| 1315 C | 298.6830 | 0.0196 | 17 | 26.5 | —10.78 | —3.5 | |
| 1316 D | 298.6871 | 0.0237 | 17 | 26.5 | — 5.29 | +0.1 | |
| 1317 B | 298.6969 | 0.0335 | 18 | 40 | + 3.34 | +4.6 | |
| 1318 C | 298.7010 | 0.0376 | 14 | 17 | — 4.78 | —4.8 | |
| 1319 D | 298.7052 | 0.0418 | 14 | 21 | + 2.42 | +1.4 | |
| 1320 A | 298.7101 | 0.0467 | 16 | 21 | + 0.69 | —1.2 | Poor. |
| 1321 B | 298.7163 | 0.0529 | 18 | 24.5 | — 0.77 | —3.0 | |
| 1322 C | 298.7212 | 0.0578 | 18 | 27 | + 1.74 | —0.2 | |
| 1323 D | 298.7260 | 0.0626 | 19 | 27.5 | + 4.14 | +2.8 | |
| 1324 A | 298.7302 | 0.0668 | 17 | 35 | — 7.15 | —7.4 | |
| 1325 B | 298.7344 | 0.0710 | 19 | 44.5 | — 3.13 | +2.2 | |
| 1326 C | 298.7385 | 0.0751 | 17 | 29.5 | — 7.19 | —4.8 | |
| 1327 D | 298.7434 | 0.0800 | 19 | 40 | — 4.48 | +0.0 | |
| 1328 A | 298.7483 | 0.0849 | 19 | 31.5 | —11.03 | —4.3 | Poor. |
| 1330 C | 298.7573 | 0.0939 | 18 | 37 | — 7.99 | +3.3 | |
| 1331 D | 298.7621 | 0.0987 | 19 | 31.5 | —19.19 | —5.6 | |
| 1332 A | 298.7670 | 0.1036 | 17 | 27 | — 8.69 | +7.6 | |
| 1333 B | 298.7746 | 0.1112 | 20 | 39 | —22.06 | —2.0 | |
| 1334 C | 298.7805 | 0.1171 | 20 | 44.5 | —29.30 | —6.8 | |
| 1335 D | 298.7941 | 0.1307 | 19 | 34.5 | —30.57 | —3.5 | |
| 1336 A | 298.7983 | 0.1359 | 8 | 17 | —24.14 | +4.1 | Poor. |
| 1337 B | 298.8031 | 0.1397 | 20 | 37.5 | —27.54 | +1.3 | |
| 1338 C | 298.8073 | 0.1439 | 19 | 42.5 | —27.60 | +1.6 | |
| 1410 C | 312.5437 | 0.1657 | 15 | 27 | —28.53 | —1.5 | Poor. |
| 1411 B | 312.5528 | 0.1748 | 18 | 27 | —24.70 | —0.4 | |
| 1412 D | 312.5576 | 0.1796 | 19 | 34.5 | —20.77 | +1.5 | |
| 1413 A | 312.5628 | 0.1848 | 20 | 27.5 | —24.82 | —4.9 | |
| 1414 B | 312.5677 | 0.1897 | 17 | 24.5 | —19.90 | —2.2 | |
| 1415 C | 312.5727 | 0.0043 | 17 | 28.5 | —18.13 | —0.9 | |
| 1416 D | 312.5788 | 0.0104 | 20 | 31 | —15.81 | —3.8 | |
| 1417 A | 312.5837 | 0.0153 | 17 | 21.5 | — 5.34 | +4.1 | |
| 1418 B | 312.5896 | 0.0212 | 16 | 30 | —11.42 | —4.9 | |
| 1419 D | 312.5944 | 0.0260 | 16 | 25 | — 1.02 | +3.2 | |
| 1420 D | 312.5996 | 0.0312 | 17 | 31 | — 4.22 | —2.2 | |
| 1421 A | 312.6045 | 0.0361 | 16 | 23.5 | — 2.27 | —1.8 | Poor. |
| 1422 B | 312.6097 | 0.0413 | 18 | 32 | + 4.99 | +4.1 | |
| 1423 C | 312.6146 | 0.0462 | 14 | 18.5 | — 1.73 | —3.6 | |
| 1425 A | 312.6246 | 0.0562 | 18 | 34.5 | + 0.69 | —1.3 | |
| 1426 B | 312.6299 | 0.0615 | 17 | 27.5 | — 3.63 | —5.1 | |
| 1427 C | 312.6399 | 0.0715 | 16 | 25.5 | — 5.58 | —4.6 | |
| 1428 D | 312.6448 | 0.0764 | 14 | 25 | — 5.14 | —2.3 | |
| 1429 A | 312.6500 | 0.0816 | 16 | 26 | —10.69 | —5.6 | |
| 1430 B | 312.6548 | 0.0864 | 13 | 17.5 | —14.22 | —6.7 | Poor. |

TABLE II. ANN ARBOR OBSERVATIONS—Continued.

| PLATE NO. | G. M. T. 1912. | PHASE. | NO. OF LINES. | WT. | VELOCITY. | O–C. | REMARKS. |
|---|---|---|---|---|---|---|---|
| | | | | | km. | km. | |
| 1431 C | 312.6601 | 0.0917 | 15 | 22 | —17.22 | —7.1 | |
| 1432 D | 312.6649 | 0.0965 | 13 | 21.5 | —14.04 | —1.4 | |
| 1433 A | 312.6701 | 0.1017 | 17 | 25.5 | —20.95 | —5.8 | |
| 1434 B | 312.6750 | 0.1066 | 15 | 24.5 | —23.18 | —5.7 | |
| 1435 C | 312.6861 | 0.1177 | 15 | 20.5 | —24.66 | —2.0 | |
| 1436 D | 312.6920 | 0.1236 | 18 | 25 | —28.50 | —3.5 | |
| 1437 A | 312.6976 | 0.1292 | 18 | 21 | —25.72 | +1.0 | |
| 1438 B | 312.7028 | 0.1344 | 19 | 25.5 | —27.85 | —0.2 | |
| 1439 C | 312.7076 | 0.1392 | 19 | 24 | —35.23 | —6.5 | |
| 1440 D | 312.7128 | 0.1444 | 16 | 20.5 | —32.45 | —3.2 | |
| 1441 A | 312.7177 | 0.1493 | 18 | 26.5 | —31.65 | —2.3 | |
| 1442 C | 312.7229 | 0.1545 | 19 | 34 | —27.08 | +1.9 | |
| 1443 D | 312.7277 | 0.1593 | 18 | 26 | —22.90 | +5.4 | |
| 1444 D | 312.7337 | 0.1653 | 14 | 19.5 | —28.58 | —1.5 | |
| 1445 B | 312.7384 | 0.1700 | 7 | 8.5 | —21.07 | +5.9 | Poor. |
| 1446 C | 312.7451 | 0.1767 | 11 | 8 | —21.57 | +2.0 | Poor. |
| 1481 B | 326.6246 | 0.1512 | 22 | 34.5 | —32.48 | —3.2 | |
| 1482 C | 326.6358 | 0.1624 | 21 | 32.5 | —25.87 | +1.9 | |
| 1483 D | 326.6469 | 0.1735 | 20 | 20 | —22.54 | +2.2 | |
| 1484 A | 326.6580 | 0.1846 | 21 | 22 | —17.48 | +2.5 | |
| 1485 B | 326.6691 | 0.0052 | 21 | 21.5 | —18.06 | —3.3 | |
| 1486 C | 326.6802 | 0.0163 | 21 | 21.5 | —16.72 | —7.6 | Poor. |
| 1487 D | 326.6913 | 0.0274 | 20 | 38.5 | —7.17 | —3.6 | |
| 1489 A | 326.7024 | 0.0385 | 19 | 32.5 | —0.05 | +0.1 | |
| 1490 | 326.7135 | 0.0496 | 17 | 26.5 | —1.84 | —4.0 | |
| 1491 C | 326.7246 | 0.0607 | 18 | 28 | +0.09 | —1.6 | |
| 1492 D | 326.7358 | 0.0719 | 18 | 27.5 | —0.69 | +0.4 | |
| 1493 A | 326.7469 | 0.0830 | 17 | 25.5 | —9.59 | —3.8 | |
| 1494 B | 326.7580 | 0.0951 | 18 | 27 | —5.20 | +6.8 | |
| 1495 C | 326.7691 | 0.1052 | 17 | 23.5 | —20.69 | —3.6 | |
| 1496 D | 326.7913 | 0.1274 | 17 | 27 | —18.14 | +8.0 | |
| 1497 A | 326.8024 | 0.1385 | 14 | 19.5 | —25.34 | +3.3 | |
| 1498 B | 326.8184 | 0.1545 | 17 | 28.5 | —25.63 | +3.4 | |

PERIOD

The period as determined by Frost was used as the basis for obtaining the period which has been used in this discussion. In obtaining the new period determinations were made of the epochs of the cross-points with the center of mass axis of the velocity curves obtained on the different nights at this Observatory. These values were compared with the epochs for the points as given by the curve drawn by Frost for July 6, 1906. The differences between the corresponding times of these determinations divided by the assumed period gave the number of cycles which had elapsed between these two dates and thereby a means was afforded for the correction of the

assumed period. The following values of the period were obtained by this method.

The mean period of 0.1904795 days was adopted and used throughout this investigation.

PERIOD DETERMINATIONS.

| DATE. | PERIOD. |
|---|---|
| August 29, 1912 | 0.1904804 days. |
| October 1, 1912 | 0.1904792 days. |
| October 1, 1912 | 0.1904796 days. |
| October 24, 1912 | 0.1904794 days. |
| October 24, 1912 | 0.1904792 days. |
| November 7, 1912 | 0.1904784 days. |
| November 7, 1912 | 0.1904796 days. |
| November 21, 1912 | 0.1904799 days. |
| November 21, 1912 | 0.1904799 days. |
| Mean | 0.1904795 days. |

NORMAL PLACES

The observed radial velocities given in column 6 of Table II were combined into normal places with phase as the basis for grouping. One hundredth of a day was the phase interval over which each normal place extended and accordingly nineteen normal places were formed. The weight for each place was determined on the basis of the sum of the weights of the plates combined to make the normal place. The largest of these sums was taken as unity and the rest reduced to this basis. The phases were reduced to the sun.

The normal places were plotted and a smooth curve was drawn through them. Correction to the normal place velocity for curvature of the curve was then investigated at the maximum and minimum points where such a correction would make itself most manifest. This correction proved to be very small and was not needed outside these regions. Within the limits where it was appreciable it was applied.

The following table gives the data for the normal places.

TABLE III. NORMAL PLACES.

| NO. | PHASE. | LIMITS OF PHASE. | VELOCITY. | O–C. | WT. | NO. OF PLATES. |
|---|---|---|---|---|---|---|
| 1 | 0.0056 | 0.00 to 0.01 | −14.48 | +0.11 | 0.522 | 7 |
| 2 | 0.0166 | 0.01 to 0.02 | −10.07 | −1.19 | 0.616 | 8 |
| 3 | 0.0269 | 0.02 to 0.03 | − 3.88 | +0.08 | 0.583 | 7 |
| 4 | 0.0367 | 0.03 to 0.04 | + 0.38 | +0.67 | 0.808 | 9 |
| 5 | 0.0464 | 0.04 to 0.05 | + 2.84 | +0.98 | 0.621 | 8 |
| 6 | 0.0572 | 0.05 to 0.06 | + 2.83 | +0.71 | 0.665 | 8 |
| 7 | 0.0648 | 0.06 to 0.07 | + 0.01 | −0.91 | 0.612 | 7 |
| 8 | 0.0748 | 0.07 to 0.08 | − 3.37 | −1.17 | 0.672 | 7 |
| 9 | 0.0850 | 0.08 to 0.09 | − 7.50 | −0.77 | 0.947 | 10 |
| 10 | 0.0973 | 0.09 to 0.10 | −11.83 | +1.24 | 0.928 | 10 |
| 11 | 0.1074 | 0.10 to 0.11 | −17.09 | +1.14 | 0.545 | 7 |
| 12 | 0.1178 | 0.11 to 0.12 | −22.57 | +0.33 | 0.622 | 7 |
| 13 | 0.1273 | 0.12 to 0.13 | −26.26 | −0.06 | 0.561 | 8 |
| 14 | 0.1380 | 0.13 to 0.14 | −30.04 | −1.48 | 1.000 | 13 |
| 15 | 0.1472 | 0.14 to 0.15 | −29.65 | −0.36 | 0.788 | 10 |
| 16 | 0.1575 | 0.15 to 0.16 | −28.43 | +0.21 | 0.809 | 9 |
| 17 | 0.1672 | 0.16 to 0.17 | −25.11 | +1.55 | 0.928 | 11 |
| 18 | 0.1768 | 0.17 to 0.18 | −22.49 | +1.02 | 0.609 | 8 |
| 19 | 0.1863 | 0.18 to 0.19 | −21.07 | −1.62 | 0.696 | 9 |

LEAST SQUARE SOLUTIONS

A definitive discussion of the orbit requires that the most probable value of $\omega$ and of $T$ be found. Both of these values can not be conveniently determined from the least square solution on account of the low eccentricity. From an intensive study of the observations in which different values of $\omega$ were assumed and the corresponding values of $T$ computed by the method of least squares the preliminary elements which are given below were adopted for the final least square solution.

PRELIMINARY ELEMENTS.

$P = 0.1904795$ days,
$T = 0^d.0543$ Phase, or
1912, August 23.8119 G. M. T.,
$e = 0.044$,
$\omega = +5^\circ .175$,
$K = 16.125$ km.,
$\gamma' = -13.38$ km.

The method used in the least square solution is essentially that outlined by Dr. Schlesinger in Volume I of the *Allegheny Publications*. Assuming the constancy of the period and the epoch of periastron and employing at Professor Curtiss's suggestion $d\gamma'$, the correction to the mean velocity, instead of $\Gamma$ as an unknown, the normal equations were formed from which the corrections to the elements were found. The normal equations are as follows:

NORMAL EQUATIONS.

$$+13.532d\gamma' - 1.083\kappa + 0.207\pi + 5.732\epsilon + 0.177 = 0,$$
$$+7.137\kappa + 0.316\pi + 0.122\epsilon + 1.965 = 0,$$
$$+6.395\pi + 0.318\epsilon - 4.499 = 0,$$
$$+3.808\epsilon - 0.121 = 0.$$

From these equations the values of the unknown quantities were found to be

$$\epsilon = +0.161,$$
$$\pi = +0.7157,$$
$$\kappa = -0.3277,$$
$$d\gamma' = -0.1185.$$

The corrections to the elements were now obtained and applied in the manner indicated. The corrections to the elements are as follows:

$$\Delta e = -0.0045,$$
$$\Delta\omega = -2^\circ .543,$$
$$\Delta K = -0.327 \text{ km.},$$
$$\Delta\gamma' = -0.1185 \text{ km.}$$

By adding these corrections to the *Preliminary Elements* we obtain the following

FINAL ELEMENTS.

$P = 0.1904795$ days,
$T = 0^d.0543$ Phase, or
1912, August 23.8119 G. M. T.,
$e = 0.040 \pm 0.015$,
$\omega = +2^\circ .632 \pm 0^\circ .917$,
$K = 15.798 \pm 0.24$ km.,
$\gamma' = -13.4985 \pm 0.21$ km.,
$a \sin i = 40{,}541$ km.,
$\gamma = -14.13$ km.

A second ephemeris was now computed on the basis of the corrected elements and the residuals formed in the usual sense. These residuals were compared with the corrected residuals of the first ephemeris. The resulting differences, the largest of which was $+0.04$ km., showed that a second solution was unnecessary. The sum of the weighted squares of the residuals was reduced from 17.10 to 13.39. The change in this quantity is small, which shows that the observed velocities were well represented by the assumed elements. The probable error of a normal place of weight unity was determined as $\pm 0.637$ km. The probable error of a single plate was $\pm 2.95$ km.

The residuals of the normal place velocities computed from the final elements are found in column 5 of Table III. A glance at these residuals reveals at once an alternation of positive and negative values, suggestive of a secondary oscillation. In Fig. 1 of Plate IX the normal place observed velocities are plotted as small circles, and the velocity curve corresponding to the final elements is drawn as a continuous line. The regular oscillation of the observed velocities above and below the curve corresponding to the final elliptic elements seems well indicated. In this figure the horizontal heavy line represents the mean velocity of the system. The line of long and short dashes oscillating about this horizontal line represents the secondary variation, which when superposed upon the elliptic velocity curve yields the curve of short dashes. This composite curve evidently satisfies the plotted velocities better than the elliptic curve.

At this point the advisability is suggested of a further least square solution containing terms

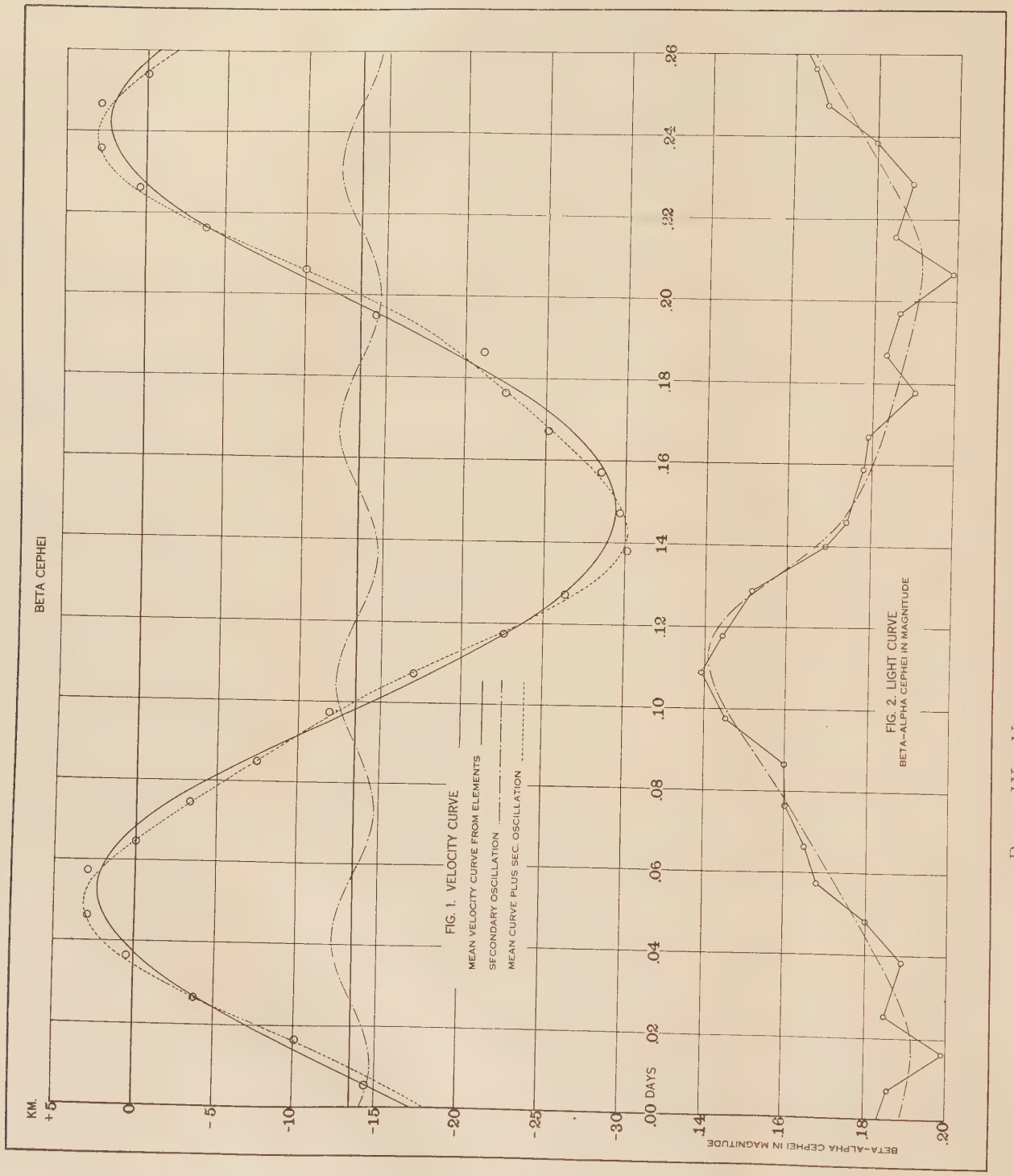


PLATE IX. VELOCITY AND LIGHT CURVES OF $\beta$ CEPHEI

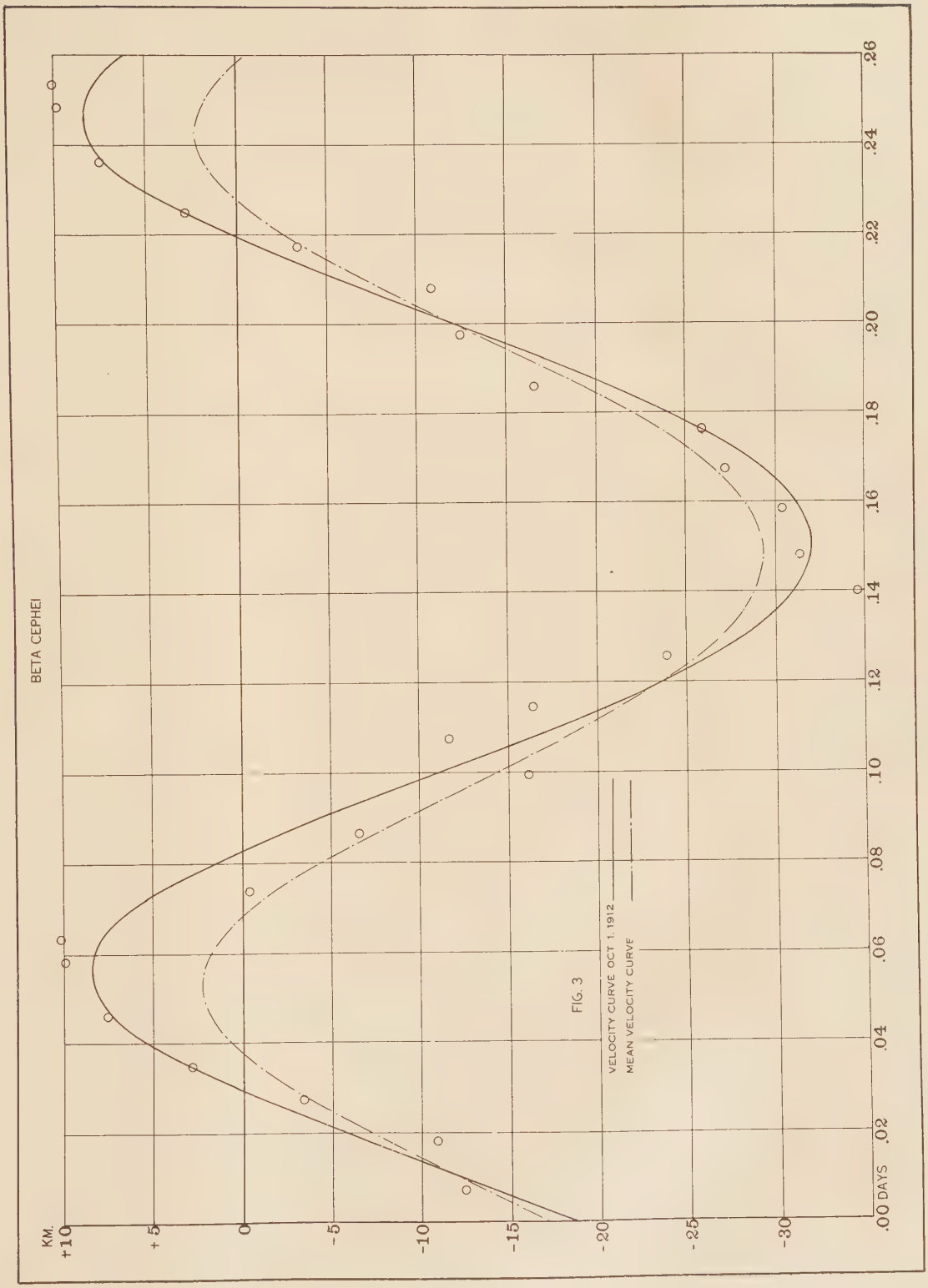


PLATE X. VELOCITY CURVE OF $\beta$ CEPHEI FOR OCTOBER 1, 1912, COMPARED WITH MEAN CURVE

determinative of the elements of the secondary oscillation. It is hoped that this solution may be made in connection with further studies of this star. For the present in drawing the curves of Fig. 1, Plate IX, after several trials in which the period of the secondary oscillation was taken to be one-third the major period, the semi-amplitude was assumed to be 1.25 km. For the present these values of the elements of the secondary oscillation may be taken as close approximations to definitive values.

OBSERVATIONS OF OCTOBER 1, 1912.

The observations made on the night of October 1, 1912, show a shift of both the points of maximum and minimum from those which were generally observed. The spectrograms secured on this night have been investigated for possible blend effects with negative results. These spectrograms seemed not to be different from those taken on other nights. The lines of the spectrum of the star seemed to be of the same average intensity and the average weight of a plate of this set was in close agreement with the average weight of a plate of a set taken on the night of October 24, which showed no variation from the generally observed results. The Observations of October 1, when plotted, (see Plate X,) seemed to be well satisfied by two straight lines. However, this may have been due to accident and motion in a conic section has been assumed in the reductions.

In order to investigate more thoroughly whether the divergence of the velocities of October 1 from the mean curve was real or due to error in measurement, or to other sources, the data were plotted in Plate X, a set of preliminary elements was computed, and a least square solution was undertaken. The assumed elements, determined by the 45° chordal point method, (see page 178 of this volume,) were as follows:

PRELIMINARY ELEMENTS.

$P = 0.1904795$ days,
$\mu = 1890° .0$,
$T = 0^d.0628$ Phase,
$e = 0.045$,
$\omega = +13°.53$,
$K = 20.31$ km.,
$\gamma' = -11.69$.

The usual method of procedure was followed, the residuals formed, and the normal equations written as indicated.

NORMAL EQUATIONS.

$$11.194 d\gamma' - 1.286_\kappa - 0.075\pi + 4.200\epsilon + 1.687 = 0,$$
$$+ 6.323_\kappa + 0.380\pi + 0.014\epsilon + 7.076 = 0,$$
$$+ 4.871\pi + 0.165\epsilon + 5.620 = 0,$$
$$+ 2.549\epsilon + 1.679 = 0.$$

From these equations the values of the unknowns were determined as follows:

$$\epsilon = -0.308,$$
$$\pi = -1.062,$$
$$\kappa = -1.088,$$
$$d\gamma' = -0.167.$$

and the corresponding corrections to the elements are

$$\Delta e = +0.007,$$
$$\Delta\omega = +2° .996,$$
$$\Delta K = -1.088 \text{ km.},$$
$$\Delta\gamma' = -0.167 \text{ km.}$$

Adding these corrections to the preliminary elements given above we get the

FINAL ELEMENTS FOR OCTOBER 1, 1912.

$P = 0.1904795$ days,
$T = 0^d.0628$ Phase, or
1912, August 23.8204 G. M. T.,
$e = 0.052 \pm 0.058$,
$\omega = +16° .526 \pm 3° .438$,
$K = 19.222 \pm 0.741$ km.,
$\gamma' = -11.857 \pm 0.744$ km.,
$\gamma = -12.88$ km.

An ephemeris was computed from these elements and the residuals formed in the usual manner. These quantities compared with the corrected residuals formed from the ephemeris based on the assumed elements showed that a second solution was unnecessary. The sum of the weighted squares of the residuals was reduced from 95.300 to 85.710. The velocity curve corresponding to these final elements is compared with the observations and the *mean curve* in Plate X.

The difference between the value of $K$ from the mean curve and that for October 1, 1912, is considerably greater than the sum of the probable errors of the determinations. The indication is that the difference is not due to accidental error and that the change in amplitude on Octo-

ber 1 was due to an unknown cause. Further investigation of the nature of this change would undoubtedly be of value.

Variations of the maxima and minima of velocity curves have been observed in the cases of $\beta$ Orionis by Plaskett and $\beta$ Coronæ Borealis by Cannon. In the case of $\beta$ Orionis, Plaskett attributes the change in amplitude to blends or to the perturbations caused by a third body. Cannon found that the shift of the maximum of $\beta$ Coronæ Borealis was periodic, repeating itself in about twelve periods. In this case he attributes the effect to "a three body system, the body which gives the spectrum measured revolving about another in 40.9 days, and the two, with a third, forming a period of perturbation of 490.8 days."

## DISCUSSION OF THE ELEMENTS

*Period.* The period of $\beta$ Cephei may be compared with those of the short period variables W Ursæ Majoris, $4^h\ 0^m$, and 14.1904 Cygni, $3^h\ 14^m$, the shortest periods known to the writer. The agreement between the length of the period observed by Frost in the spring of 1906 and that determined at this Observatory in the fall of 1912 is interesting. We can assume the value of $4^h\ 34^m\ 17^s.4$ to be, on the average, the period for the last six years.

*Eccentricity.* If we apply the probable error in the most favorable way to the value of the eccentricity as given by the definitive elements, we find an agreement of this element with that given by Frost for July 6, 1906.

| FROST, 1906. | CRUMP, 1912. |
|---|---|
| $e = 0.03 +$ | $0.01 + 0.015$. |

These values indicate that the orbit is nearly circular and the agreement shown above would indicate constancy of form.

*The Angular Distance from Periastron.* A considerable degree of uncertainty pertains to the value of $\omega$. Possibly the large difference between Frost's value of 165° 13′ and that of 2°.63 determined here is due to the uncertainties connected with the small eccentricity.

*The Time of Passage through Ascending Node.* The value of $T$ depends upon $\omega$ and is therefore difficult to fix in a nearly circular orbit. In this discussion it has been found to be 1912 August 5.8119 G.M.T.

*Velocity of the System.* The two determinations of the element $\gamma$ by Frost and that which I made are as follows:

| | | |
|---|---|---|
| July 6, 1906 | — 4. 9 km. | Frost. |
| April 29, 1912 | — 10. 2 km. | Frost. |
| August 5, 1912 | — 14.13 km. | Crump. |

The values indicate a change in the center of mass velocity. A variation of this element has been established in the case of Polaris and suspected by Belopolsky in the case of $\delta$ Cephei. In no other cases of Cepheid variables are observations concerning this variation available for study. The change in the velocity of the center of mass may possibly be attributed to the action of a third body.

*The Semi-Amplitude.* That the half amplitude of the velocity curve $K$ is subject to variation was shown by the variant curve observed on October 1, 1912. During the series of observations this was the only instance of large variation observed. If we compare the value as determined in 1906, of 16.9 km., with the value as obtained at this Observatory in 1912, of 15.8 km., we observe a fair agreement. Though this element may vary from cycle to cycle we do not know whether the average value of this quantity is changing.

## SUMMARY AND CONCLUSION

1. As a result of this investigation, definitive elements of the orbit of $\beta$ Cephei have been determined from 163 velocity observations, whose average probable error determined from an elliptic curve was found to be $\pm$ 2.95 km.

The period of $4^h\ 34^m\ 17^s.4$ is the shortest period which has thus far been observed for a spectroscopic binary. The agreement between the value determined by Frost in 1906, and that determined by the writer in 1912, to the sixth decimal place in days, indicates that on the average the period remained constant during the interval of six years.

The eccentricity of 0.040 shows that the orbit is almost circular. This element is in agreement with the general observation of astrophysicists

that spectroscopic binaries of early type have small eccentricities. It is the smallest eccentricity that has been determined for a variable of the δ Cephei type.

In nearly circular orbits the value of the angular distance of periastron is more or less uncertain. In this case the eccentricity is so small that both $\omega$ and $T$ can not be easily determined from a least square solution, and consequently, different values of $\omega$ were chosen and the corresponding times of periastron passage computed. The value of this element which seemed the best determined was used with the corresponding value of $T$ in the preliminary elements. Assuming this value of $T$ as fixed, a final least square solution gave the corrected value of $\omega$ as $+ 2°.63$. This determination seems as good as could be expected. The large difference between this value and that of Frost in 1906 may be due to uncertainties affecting its determination.

2. The observations indicate a secondary oscillation whose period appears to be about one-third of the primary. Such an oscillation has been observed by Professor Curtiss in W Sagittarii, and a similar one, though reversed, has been found by Campbell in the case of ζ Geminorum. At present the cause of such oscillations is not known, although Professor Curtiss has suggested a possible explanation in these *Publications*, Vol. I, p. 109. The presence of a secondary oscillation may prove to be a further indication of variation of the δ Cephei type.

3. The velocity curve determined by the writer together with the light curve observed at the Berlin-Babelsberg Observatory confirms Guthnick's conclusion that β Cephei is a variable star of the δ Cephei type. The disagreement observed of about one-seventh of the period between the points of light maximum and maximum velocity of approach is the same as that observed in Y Sagittarii, but occurs on the descending instead of the ascending branch of the velocity curve. This variation may possibly be connected with the closeness of the components of the system. The time interval between the minimum of light and the minimum velocity of approach is observed to be about one-fifth of the period. Although this disagreement is large it would not necessarily exclude it from the δ Cepheid class of variables. In the cases so far observed the agreement between the points of minimum light and minimum velocity of approach has not been so close, on the average, as that between the maximum of light and the maximum velocity of approach. The disagreement may be due to variations caused by the ellipsoidal form of the members of the system, or by other conditions related to the extremely short period.

4. The observations made at this Observatory indicate, as does the unpublished observation of April 29, 1912, kindly communicated to me by Professor Frost, a shift of the center of mass velocity from that determined on July 6, 1906. Only in three cases of Cepheid variables, β Cephei, Polaris, and δ Cephei, are the data available for the determination of the existence of a variation of the velocity of the center of mass. In the first two cases named the variation is present and in the third its presence is suspected. Further observations would be of interest to determine whether this is a characteristic of variables of the δ Cephei type.

5. A variation in the amplitude of the velocity curve was detected and studied. While the amplitude has varied in one case at least we do not know that the average value of this quantity is changing.

It is with pleasure that I acknowledge my indebtedness to Professor R. H. Curtiss for help and supervision throughout this investigation, to Professor G. W. Hess for aid in securing the spectrograms, and to Doctor Laurence Hadley for valuable assistance.

Detroit Observatory, Ann Arbor, Michigan, August, 1915.